ISE 7200 – Advanced Nonlinear Optimization

Spring 2022

(Last updated January 10, 2022)

1 Course Overview

Lectures: Tuesdays and Thursdays, 2:20–3:40pm.
Location: Baker Systems 128 + online via CarmenZoom. See links on CarmenCanvas page.

Instructor: Parinaz Naghizadeh
Email: naghizadeh.1@osu.edu
Office hours: Tuesdays 11:30am-12:30pm and Thursdays 4-5pm, or by appointment (please try to email me at least 2 days in advance so we can find a time).
Online via CarmenZoom. See links on CarmenCanvas page.

Catalog description: Unconstrained and constrained nonlinear optimization, covering applications, theory dealing with convexity, optimality conditions, duality, and algorithms. Prereq: Calculus, linear algebra, computer programming, and an introductory optimization course, or permission of instructor. Not open to students with credit for both 820 and 821. Units: 3 credit hours.

Informal description: This course exposes students to the fundamentals of nonlinear optimization theory and related algorithms. As this is an advanced course, the goal is to cover the topics below in depth, and hence the topics covered later in the course may be adjusted as we progress. The broad areas to be covered are:

- Optimality conditions for unconstrained and constrained optimization
- Gradient and Newton methods, and convergence properties
- Lagrange multipliers and associated necessary and sufficient conditions
- Duality theory and convex programming
- Algorithmic methods for unconstrained and constrained optimization, including steepest descent, Newton methods and variants, conjugate direction methods, interior point methods, and penalty and barrier methods

A tentative schedule of topics to be covered in each lecture is available at the end of the syllabus.

Prerequisites: Prior exposure to optimization and its applications is helpful, but not necessary. The students taking this course must have good knowledge of linear algebra and advanced calculus. This will provide the mathematical maturity to follow and appreciate the proofs and reasoning behind the algorithms covered in our course. Knowledge of, or willingness to learn, programming in Matlab, Python, R, Julia, or other similar languages, will be needed for the coding homework.

2 Textbooks, Readings, and Course Technologies

Readings and reference books: The course will primarily follow the textbook "Nonlinear Programming" (3rd ed.) by Dimitri P. Bertsekas. Having a copy will be helpful, but is not required. Other recommended references for the topics covered in the course are:

- Convex optimization, by S. Boyd and L. Vandenberghe. Cambridge University Press, 2004. *Available online at* `https://web.stanford.edu/~boyd/cvxbook/`.
- Linear and nonlinear programming, by D. G. Luenberger and Y. Ye, 2015.
- Numerical Optimization, by J. Nocedal and S. Wright. Springer, 2006.

Lecture slides (and annotated versions) will be posted on Carmen. These are intended to provide an overview of all topics covered in each lecture, but will not be a substitute to lectures, as they may not include all details such as some of the discussions or examples in class.

Copyright disclaimer: The materials used in connection with this course may be subject to copyright protection and are only for the use of students officially enrolled in the course for the educational purposes associated with the course. Copyright law must be considered before copying, retaining, or disseminating materials outside of the course.

Course technologies: For remote participation in office hours, you need to have access to a computer with a microphone, and preferably a webcam. You are probably all fairly familiar with the use of CarmenCanvas and CarmenZoom. To submit your homework and exams, you will upload the assignments on CarmenCanvas. For written homeworks, you will need access to a camera (e.g. a phone camera) to take pictures and upload them on Canvas (you do not need to use a scanner or anything fancy, anything that produces a legible image will do).

3 Grading

Grading: 30% Homework, 30% Midterm Exam, and 40% Final Exam.

- **Homework:** Homework will be assigned roughly on a biweekly basis and will be due in one week, by the end of the day. There will be a total of 6 homework sets. The tentative dates on which homework will be due are denoted in the lecture outline. You have a total of *3 late days* throughout the semester when turning in your homework. Once you have used up this allocation, late homework will be accepted only if there is a documented reason or unexpected circumstances (verifiable by the university). *No exceptions will be made; late homeworks beyond this will not be accepted.* Some homework problems will involve coding, for which you can use your programming language of choice.
- **Midterm exam:** There will a take-home midterm exam, due Tuesday, March 1st. There will be no class on that day to allow you extra time to work on the exam. The midterm will be made available after our class on Thursday 2/24. It will be based on topics covered up to and including lecture 13 (equivalent to the sections covered from chapter 1-4 of the textbook).
- **Final exam:** There will a take-home final exam, assigned on the day of our last lecture. It will be due in one week, by the end of the day, Thursday, April 28th.

The above dates are subject to change. Please refer to the course announcements on Carmen to remain notified of any updates to the syllabus, including updates to homework or exam(s) assignment and/or due dates, as well as clarifications/corrections to assignment questions.

4 Course Outline

Table 1: Lecture outline (subject to change)

Lecture	Date	Topics covered
1	1/11	Introduction and applications of nonlinear optimization
2	1/13	Mathematical background review
3	1/18	Unconstrained optimization: optimality conditions
4	1/20	Unconstrained optimization: optimality conditions, continued
5	1/25	Gradient methods
6	1/27	Convergence analysis of gradient methods
	1/28	**Homework 1 due**
7	2/1	Newton's method
8	2/3	Additional methods for unconstrained optimization problems:
9	2/8	Optimization over convex sets: optimality conditions and example
10	2/10	Feasible direction methods: Conditional gradient, Gradient projection
	2/11	**Homework 2 due**
11	2/15	Constrained optimization: Lagrange multiplier theory
12	2/17	Constrained optimization: Lagrange multiplier theory – continued
13	2/22	Lagrange multiplier theory: sufficiency conditions
	2/23	**Homework 3 due**
14	2/24	Lagrange multiplier theory: sufficiency conditions and sensitivity analysis
15	3/1	**Midterm take-home exam due** – no class to give you extra time to work on it
16	3/3	Lagrange multiplier theory: inequality constraints, KKT
17	3/8	Lagrange multiplier theory: inequality constraints – continued
18	3/10	LMT review and wrap-up
	3/11	**Homework 4 due**
	3/15 & 3/17	Spring Break (no class)
19	3/22	Barrier and interior point methods
20	3/24	Penalty methods
21	3/29	Augmented Lagrangian methods
22	3/31	Lagrange multiplier algorithms wrap-up
	4/1	**Homework 5 due**
23	4/5	Convex analysis review
24	4/7	Introduction to duality theory
25	4/12	Duality theory continued
26	4/14	Strong duality
	4/15	**Homework 6 due**
27	4/19	Duality wrap up
28	4/21	Review and pointers to additional/advanced topics
		Final take-home exam (Due end of the day, Thursday, April 28th)

5 Other Course Policies

Collaboration policy: Students are encouraged to work together on homework and on general discussion of course material. Keep in mind however that all written solutions and code handed in by the students must be written solely by them and reflect their independent understanding of the material. No collaboration, discussions, or use of any online resources, is permitted for the midterm and final exams.

Health and safety requirements: All students, faculty and staff are required to comply with and stay up to date on all university safety and health guidance (`https://safeandhealthy.osu.edu`), which includes wearing a face mask in any indoor space and maintaining a safe physical distance at all times. Non-compliance will be warned first and disciplinary actions will be taken for repeated offenses.

Academic misconduct statement: The Ohio State University and the Committee on Academic Misconduct expect that all students have read and understand the University's Code of Student Conduct, and that all students will complete all academic and scholarly assignments with fairness and honesty. Students must recognize that failure to follow the rules and guidelines established in the University's Code of Student Conduct and this syllabus may constitute "Academic Misconduct," as defined in the University's Code of Student Conduct (Section 3335–23–04). Any student found to have engaged in academic misconduct will be subject to disciplinary action by the university. Please contact me if you have any questions about what might constitutes academic misconduct in this course.

Statement on Title IX: All students and employees at Ohio State have the right to work and learn in an environment free from harassment and discrimination based on sex or gender, and the university can arrange interim measures, provide support resources, and explain investigation options, including referral to confidential resources. If you or someone you know has been harassed or discriminated against based on your sex or gender, including sexual harassment, sexual assault, relationship violence, stalking, or sexual exploitation, you may find information about your rights and options at `titleix.osu.edu` or by contacting the Ohio State Title IX Coordinator at titleix@osu.edu. Title IX is part of the Office of Institutional Equity (OIE) at Ohio State, which responds to all bias-motivated incidents of harassment and discrimination, such as race, religion, national origin, and disability. For more information on OIE, visit `equity.osu.edu` or email equity@osu.edu.

Your mental health: As a student you may experience a range of issues that can cause barriers to learning, such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you find yourself feeling isolated, anxious or overwhelmed, please know that there are resources to help: `ccs.osu.edu`. You can reach an on-call counselor when CCS is closed at (614) 292-5766 and 24 hour emergency help is also available through the 24/7 National Prevention Hotline at 1-(800)-273-TALK or at `suicidepreventionlifeline.org`. The Ohio State Wellness app is also a great resource available at `go.osu.edu/wellnessapp`.

Accessibility accommodations for students with disabilities: The University strives to make all learning experiences as accessible as possible. In light of the current pandemic, students seeking to request COVID-related accommodations may do so through the university's request process, managed by Student Life Disability Services. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately (within the first two weeks of the course) so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; `slds.osu.edu`; 098 Baker Hall, 113 W. 12th Avenue. Information about accessibility within CarmenCanvas and CarmenZoom is avilable in the following links: Carmen Canvas and Carmen-Zoom.